Interfacial instability of thin viscoelastic liquid films

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Abstract

We study the linear stability of a thin viscoelastic liquid film under the influence of van der Waals interaction. The Jeffreys model is used to describe the viscoelasticity effects with a relaxation time and a retardation time. We study the governing thin film approximation that describes the nonlinear evolution in time of the interface in the long-wave limit. We are interested in the instability that causes the dewetting of the liquid on a substrate. We include the dewetting effect through the van der Waals attractive-repulsive force. The analysis is also carried out considering the fluid in the transition from weak to moderate to strong slip regimes. In each regime, the role in the break-up of the liquid viscoelasticity as well as the contact angle or the slippage are studied. Numerical simulations of the nonlinear model are implemented and compared with the theoretical results given by the linear stability analysis.

Introduction

We simplify the generalized Maxwell model of Jeffreys type for the moving interface of viscoelastic liquids in the $2D$ lubrication approximation. This model describes the non-Newtonian nature of the stress tensor interpolating a purely elastic and a purely viscous behavior, characterized by two time constants λ_1 and λ_2 respectively, namely *relaxation time* and *retardation time*. We study the effects of the perturbation of a thin film of fluid in the presence of van der Waals forces. Our investigations are motivated by applications of thin polymer films as in semi-conductors, solar cells, etc. We carry out our analysis in regimes that transit from weak to strong-slip and see how the slippage together with the viscoelasticity affect the instability. A thin film of fluid of constant initial thickness h_0 is perturbed and the linear stability analysis on the governing equation describes whether the film breaks up into separate rims (instability) or returns to its initial configuration (stability). We drive numerical simulations of the highly non linear model in the case of no slip or weak-slip and absence of viscosity and compare them with the theoretical analysis.

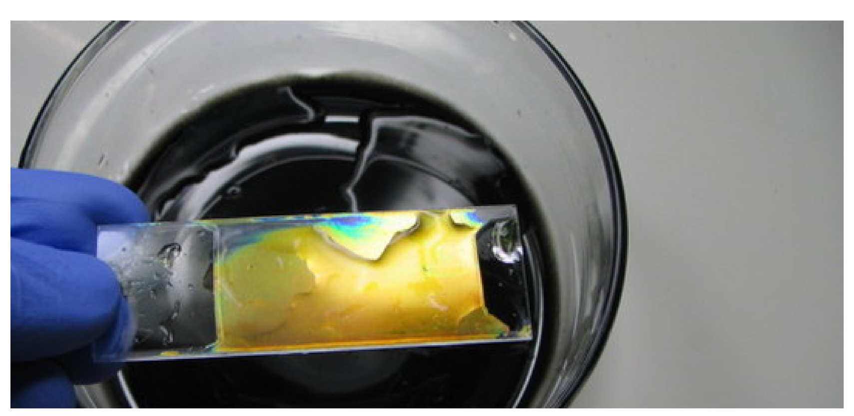

Figure 1: *An application of a thin polymeric liquid film.* Picture from www.foodproductiondaily.com - MIT research.

Governing Equations

The equation governing the hydrodynamics for the fluid interface of viscoelastic media is derived as a long-wave approximation of the conservation laws. The liquid is considered incompressible, with mass density ρ. The equation of conservation of mass and continuity of momentum are:

$$\nabla \cdot \mathbf{u} = 0\,,$$

$$\rho\left(\partial_t \mathbf{u} + \mathbf{u}\cdot\nabla\mathbf{u}\right) = -\nabla p_R + \nabla\cdot\tau\,,$$

where $\mathbf{u} = (u, v, w)$ is the velocity vector field, and p_R the reduced pressure such that $p_R = p - \Pi$, where p is the hydrostatic pressure, while Π is the pressure induced by body forces of van der Waals type (attractive or repulsive). The stress tensor τ follows the Jeffreys model for viscoelastic fluids, which describes the nonlinear relation $\tau(\dot\gamma)$ between the stress tensor τ and the *strain rate* $\dot\gamma$:

$$\tau + \lambda_1\partial_t\tau = \eta(\dot\gamma + \lambda_2\partial_t\dot\gamma) \tag{1}$$

in which η is the shear viscosity coefficient and λ_1, λ_2 are the two relaxation times of the liquid when it shrinks back to its original shape after deformation, with generally $\lambda_1 > \lambda_2$.
In figure 2 we can see a scheme of the fluid's interface. At the solid substrate we have Navier boundary conditions where $b \geq 0$ is the *slip length* ($b = 0$ means no slip, and $b \gg 1$ means strong-slip).

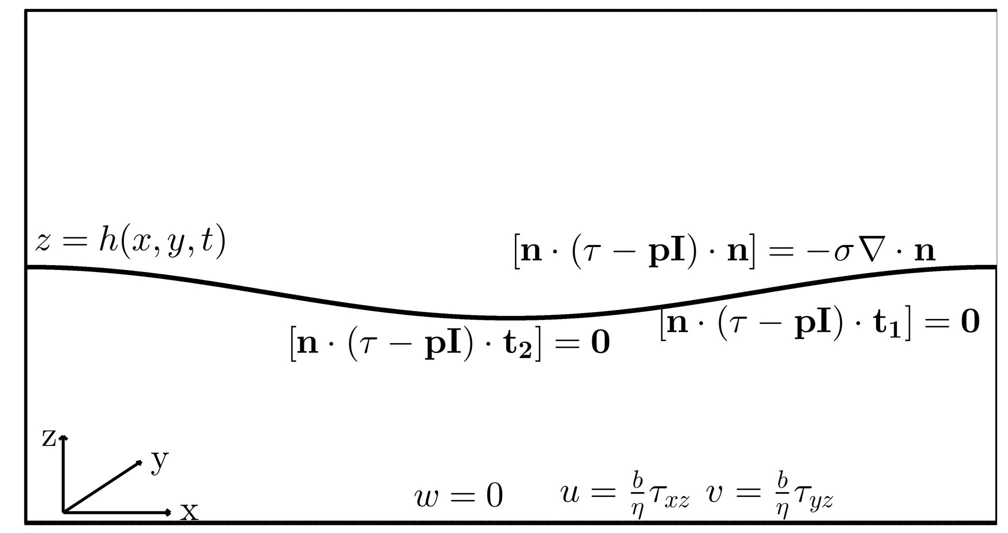

Figure 2: *Scheme of the fluid interface and boundary conditions.*

We nondimensionalize these equations, using:

$$(x, y) = L(x^*, y^*),\ z = Hz^*,\ (u, v) = U(u^*, v^*),\ w = \varepsilon U w^*$$

$$t = Tt^*\,,\ \text{with}\ T = \frac{L}{U}\ \text{and}\ \frac{H}{L} = \varepsilon\,,$$

where ε is the small parameter. In the weak-slip regime the slip length $b = O(1)$ and the pressure is scaled as $PH/\eta U \sim \varepsilon^{-1}$ [4].
Keeping only $O(1)$ terms in the boundary conditions we obtain $p_R = -\nabla^2 h - \Pi$, and using this together with the kinematic boundary condition into the governing equations (dropping the *) leads to the closed form equation for the fluid's interface:

$$(1+\lambda_2\partial_t)h_t + (\lambda_2 - \lambda_1)\nabla\cdot\left[\left(\frac{h^2}{2}\mathbf{Q} - h\mathbf{R}\right)h_t\right] =$$
$$\nabla\cdot\left\{\left[(\ +\lambda_1\partial_t)\frac{h^3}{3}\nabla p_R + (1+\lambda_2\partial_t)bh^2\nabla p_R\right]\right\} \tag{2}$$

where $\mathbf{Q}$ and $\mathbf{R}$ satisfy

$$\mathbf{Q} + \lambda_2\mathbf{Q}_t = \nabla p_R,\ \ \mathbf{R} + \lambda_2\mathbf{R}_t = h\nabla p_R$$

and the van der Waals potential is defined by:

$$\Pi(h) = \frac{\sigma(1 - cos\theta)}{Mh_\star}\left[\left(\frac{h_\star}{h}\right)^n - \left(\frac{h_\star}{h}\right)^m\right],$$

with θ the contact angle, $M = (n-m)/[(m-1)(n-1)]$ (generally $n > m$) [2], and σ the surface tension.

Linear Stability Analysis

To study the film's response to a perturbation we consider $h = h_0 + \delta h_0 e^{ikx+\omega t}$, $Q = \delta Q_1$, $R = \delta R_1$, where h_0 is the flat initial thickness, k the wave number $k = 2\pi/\lambda$, and ω the growth rate. Using these into equation (2) and keeping only terms up to $O(\delta)$, we obtain the following disperion/dissipation relation:

$$\lambda_2\omega^2 + \left[1 + (k^4 - k^2\Pi'(h_0))\left(\lambda_1\frac{h_0^3}{3} + \lambda_2 bh_0^2\right)\right]\omega + (k^4 - k^2\Pi'(h_0))\left(\frac{h_0^3}{3} + bh_0^2\right) = 0\,. \tag{3}$$

Solving for the two roots of this quadratic equation we obtain one root strictly negative, let us say ω_2, and one root with varying sign, call it ω_1. The latter one is positive (unstable) for $-\sqrt{\Pi'(h_0)} < \omega_1 < \sqrt{\Pi'(h_0)}$. The most unstable mode is given by $k_m = \pm\sqrt{\Pi'(h_0)/2}$. Therefore from the definition above we can see that both $k_c = \pm\sqrt{\Pi'(h_0)}$ and k_m do not depend on the viscoelasticity times λ_1 and λ_2 and neither on the slip length b.

Numerical Results

We developed simulations of the evolution of the film in the $1D$ case of absence of viscosity, where the equation (2) governing the motion of the free surface of the liquid reduces to:

$$h_t - \frac{\lambda_1}{2}\frac{\partial}{\partial x}\left[h^2\left(h_{xxx} + \Pi'(h)h_x\right)h_t\right]$$
$$+\frac{\partial}{\partial x}\quad\left[\left(\frac{h^3}{3} + bh^2\right)\left(h_{xxx} + \Pi'(h)h_x\right) + \lambda_1\frac{\partial}{\partial t}\left(\frac{h^3}{3}\left(h_{xxx} + \Pi'(h)h_x\right)\right)\right] = 0\,.$$

The numerical method uses Newton linearization of the nonlinear term and (implicit) Crank-Nicolson and central finite differences for the spacial derivatives [3].

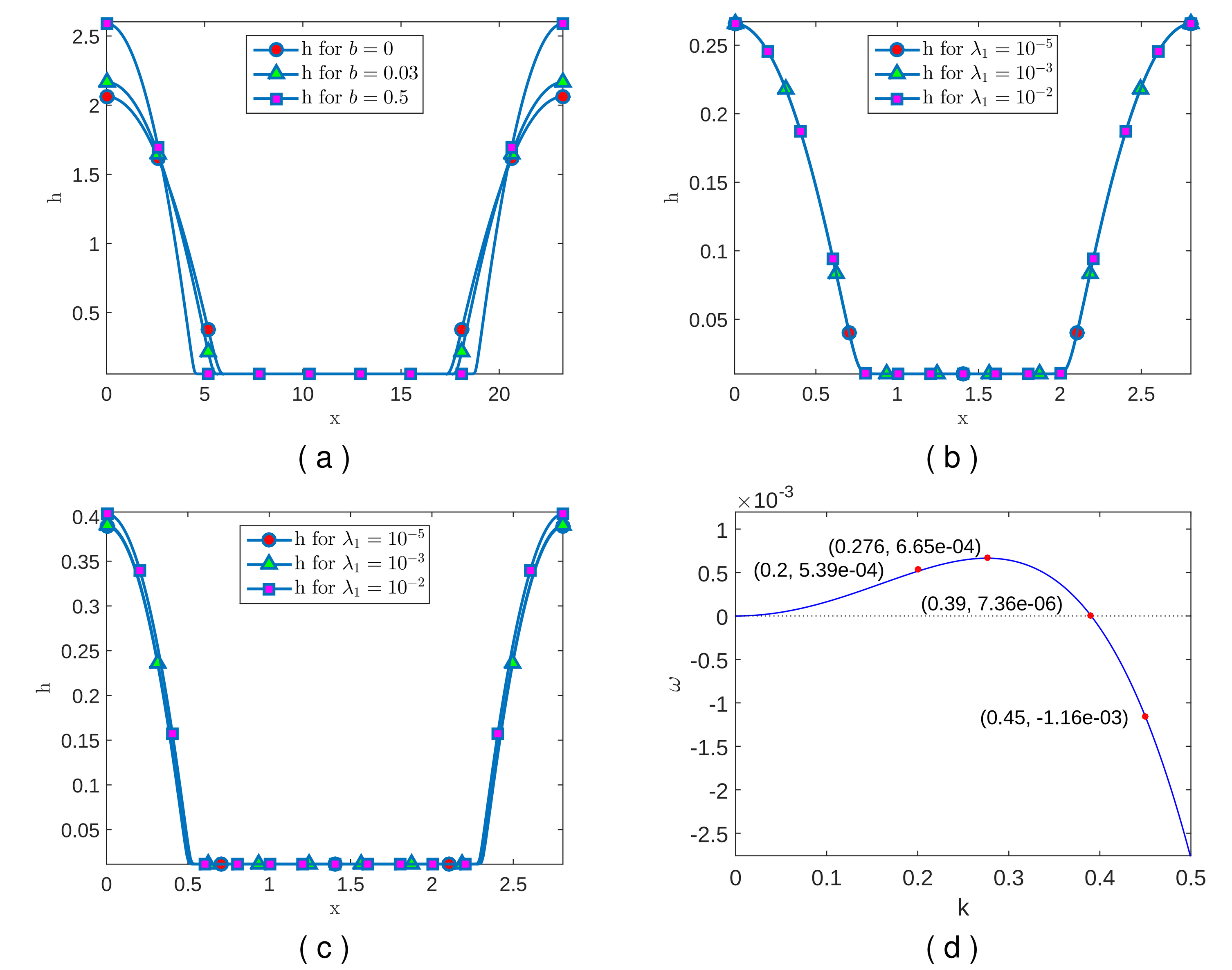

Figure 3: *(a), (b) and (c): Fluid's interface instability - transition from no slip to weak-slip regime with different values of relaxation times -; (d): Growth rate of interfacial instability given by the numerical simulations compared with the theoretical LSA.*

In figure 3(a) we see the evolution in time of a film of initial thickness $h_0 = 0.1$: the liquid's interface is perturbed and it does not returns to its flat profile, but it breaks up into two separate rims. The instability is due to van der Waals forces' interaction with a precursor film $h_\star = 0.01$ and contact angle $\theta = 45°$. In figure 3(b) and 3(c) we see the final film's interface configuration for different values of *relaxation time* λ_1 for fixed initial height h_0 and $h_\star = 0.01$ and $b = 0$ and $b = 0.1$ respectively. In figure 3(d) instead we compare the growth rates of the instabilities for different wave lengths with the theoretical results given by the Linear Stability Analysis.

Conclusions and Future Work

The numerical results of our simulations are in agreement with the linear stability analysis. In our future work we will implement the full nonlinear equation (2) in the weak-slip regime and investigate how the transition from weak to moderate to strong-slip regimes affects the instability together with the viscoelastic effects.

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